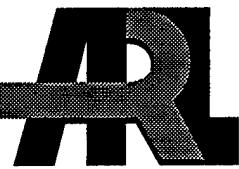


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# **Statistical Analysis of Surface Pressure Measurements vs. Computational Predictions**

by Malcolm S. Taylor  
and Walter B. Sturek

ARL-TR-1318

March 1997

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**March 1997**

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## **Statistical Analysis of Surface Pressure Measurements vs. Computational Predictions**

**Malcolm S. Taylor, Walter B. Sturek**  
Information Science and Technology Directorate, ARL

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## **Abstract**

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This report details a statistical approach to the comparison of results from an international study in which Navier-Stokes computational techniques were applied to a complex flow field with highly separated flow about a fixed geometry. The traditional method for evaluating flow-field computations is for the engineer to visually inspect the results, make qualitative comparisons, and draw conclusions—a process that can be effective for a limited number of data sets. In this study, the number of data sets was so large as to make the evaluation and comparison of the results a formidable task. In response, a statistical approach with emphasis on data visualization was developed. The procedure extends directly to higher dimensions, and is of particular value when a large number of data sets are to be compared and an impartial quantitative assessment is sought.

## **ACKNOWLEDGMENTS**

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## 1. INTRODUCTION

This report discusses the development and application of a statistical analysis approach to the evaluation of some results of a collaborative study carried out under the auspices of The Technical Cooperation Program (TTCP) with participants from Canada, the United Kingdom, and the United States. The purpose of the study was to apply Navier-Stokes computational techniques to a complex flow field with highly separated flow about a fixed missile geometry to evaluate the predictive technology.

The computational study encompassed 8 computer codes, 9 turbulent viscous models, several laminar viscous results, as many as 15 grid resolutions, and 6 test cases. This produced more than 90 test-case results for evaluation with respect to each other and to effects of grid resolution, choice of turbulence model, and computational technique.

A Schlieren photograph showing the test model mounted in the wind tunnel for a Mach 3.5 flow condition and 8-deg angle of incidence is shown in Figure 1. The bow shock and lee-side separated vortical flow are clearly visible. In Figure 2, experimental measurements of surface pressure taken at eight axial locations,  $x/d$ , are plotted along with the computed values from laminar and turbulent viscous models. The computed results show generally close agreement with the experiment on the windward side ( $0 \leq \phi \leq 90$  deg) and disagreement on the leeward side ( $90 \leq \phi \leq 180$  deg). The lee-side separated flow is the critical region for evaluating the predictive capability of the computational procedures.

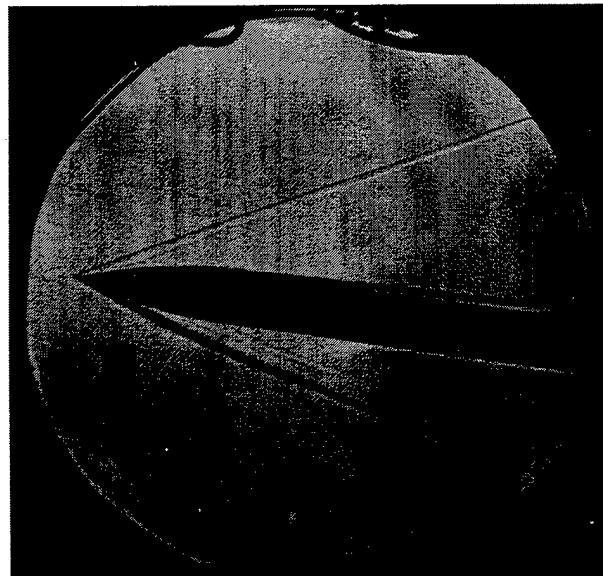


FIGURE 1. *Schlieren photograph of model mounted in wind tunnel; Mach 3.5, angle of incidence 8 deg.*

The traditional method for evaluating flow-field computations is for the engineer to visually inspect the results, make qualitative comparisons, and draw conclusions—a process that can be quite satisfactory for a limited number of data sets. However, for this study, the number of data sets was so large as to make the evaluation and comparison of the results a formidable task, and it was decided to explore the contribution that statistical data analysis techniques might be able to make in the evaluation process.

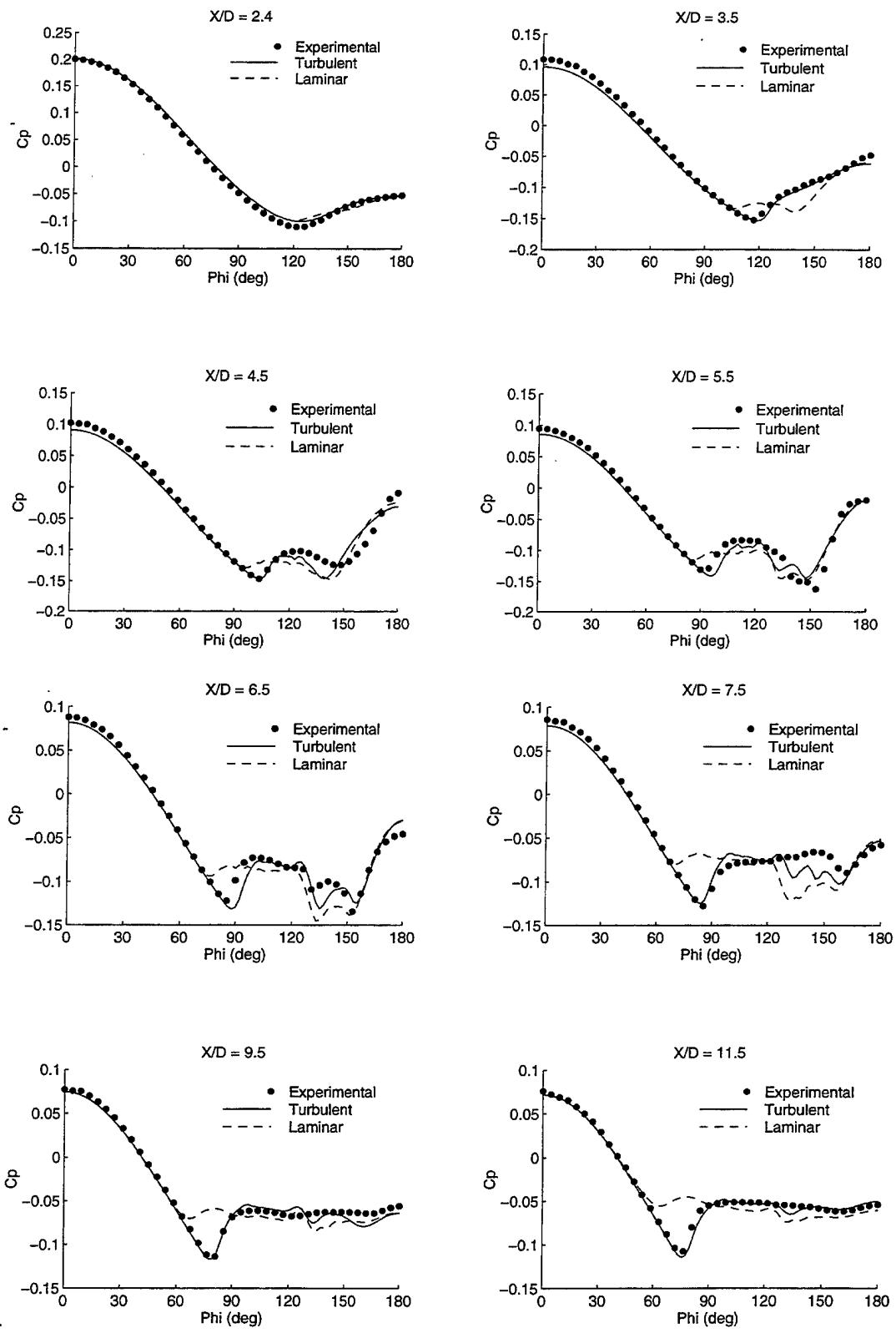


FIGURE 2. Surface pressure  $C_p$  vs. circumferential position  $\phi$ ; Mach 2.5, angle of incidence 14 deg. Comparison between computation and experiment.

## 2. STATISTICAL APPROACH

The wide availability of powerful and affordable computing resources has impacted the way in which data analysis is conducted at a fundamental level, and expressions such as “exploratory data analysis” and “data visualization” have made their way into the statistical lexicon. Visualization of quantitative data is an active area of research, with professional journals and books devoted exclusively to the topic (Tufte 1983, 1990; Cleveland 1993; DuToit, Steyn, and Stumpf 1986). The basic idea is that data, if properly collected and effectively portrayed, can often speak directly to the subject-matter expert without resorting to a plethora of statistical jargon and assumptions. This is the approach undertaken in this section; a subset of TTCP test-case results is used for illustrative purposes.

### 2.1 Data Structure

A set of experimental data was chosen to serve as a baseline against which all the flowfield predictors would be compared. The experimental data pertinent to this investigation consist of measurements of surface pressure taken at 41 circumferential positions,  $\phi$ , about each of 9 axial locations,  $x/d$ , for a test model such as that shown in Figure 1.

The technique involves the comparison of two data sets—the experimental (or baseline) data and a set of computed results. Both data sets are three-dimensional, consisting of the triples: (axial location, circumferential position, surface pressure). To facilitate discussion, the experimental data are denoted as

$$Y_{ij} \quad i = 3.5(1)11.5, \quad j = 0(4.5)180, \quad (1)$$

where the subscript  $i$  indexes axial location,  $x/d$ ; the subscript  $j$  indexes circumferential position,  $\phi$ ; and  $Y_{ij}$  is the measured surface pressure. The computed value is denoted as

$$X_{ij}^k \quad k = 1(1)9, \quad (2)$$

where subscripts  $i$  and  $j$  are the same as for the experimental data; the superscript  $k$  indexes the data sets chosen for analysis.

### 2.2 Error Analysis

If we define the difference between a computed value of surface pressure and the corresponding experimental value as error, then

$$E_{ij}^k = X_{ij}^k - Y_{ij} \quad (3)$$

is the error at location  $(i, j)$  for data set  $k$ . A three-dimensional contour plot provides an informative display of error over the entire axial-circumferential grid; such a plot for the data set corresponding to  $k = 1$  is shown in Figure 3.

The approximately planar surface in Figure 3 represents the windward region where experimental and computed values are in reasonably good agreement. The projection of the surface onto the horizontal plane further aids in the definition of this region; for data set  $k = 1$ , the substantial irregularities occur on the leeward side, corresponding to values of  $\phi$  greater than 90 deg.

The highly irregular contour surface in Figure 3 provides convincing evidence of the need for further data reduction before the simultaneous comparison of several flowfield calculations can be

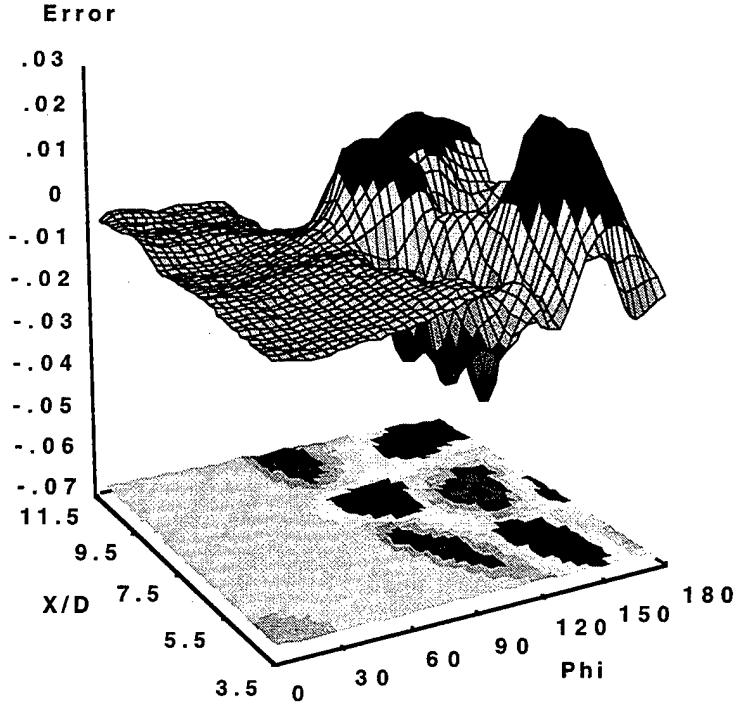


FIGURE 3. Contour surface of error for data set  $k=1$ .

effectively undertaken. A further data summary using box and whisker plots is shown in Figure 4. To construct a box and whisker plot, the errors over the entire grid are combined and ranked in value from smallest to largest. The median, or middle, value is visually represented by the dot ( $\bullet$ ) inside the boxes in Figure 4; the top and bottom of the box mark the 75th and 25th percentiles, respectively, of the ranked errors; the whiskers are vertical lines that terminate at the extreme values of the data set. The width of the box holds no significance. These five-number data summaries provide a compact description of how the errors are distributed over the entire grid, and, as can be seen, facilitate comparison across several data sets.

The perfect case, where computed and experimental values coincide at every location  $(i,j)$ , would cause the box and whisker plot to collapse to a single value—zero. Failing that, a thin box with short whiskers indicates good overall agreement. A cursory glance at Figure 4 suggests that data sets 3 and 4 may be closest to this ideal, while data set 8 appears in some difficulty. Since the whiskers terminate at the maximum and minimum error values, a spurious, or outlying, observation could distort the whisker length and mislead the viewer. The data sets in Figure 4 were inspected and found to be free from outliers—a precautionary procedure that should be routinely performed.

The statistical graphics in Figures 3–4, while enlightening, suggestive, and highly appropriate for an initial screening, are not entirely adequate for an impartial assessment of which error sets represent “closest” agreement between experiment and model. Clearly, we would like the errors to be tightly clustered about zero. This would be reflected in a location parameter (a mean or median) and a dispersion parameter (a standard deviation or interquartile range) both close to zero. This suggests that an attempt to formally rank the effectiveness of the computation procedures as revealed through the error sets  $\{E_{ij}^k\}$ ,  $k = 1, 2, \dots, 9$ , should involve a statistic that includes a measure of both location and dispersion. And this is exactly what was done.

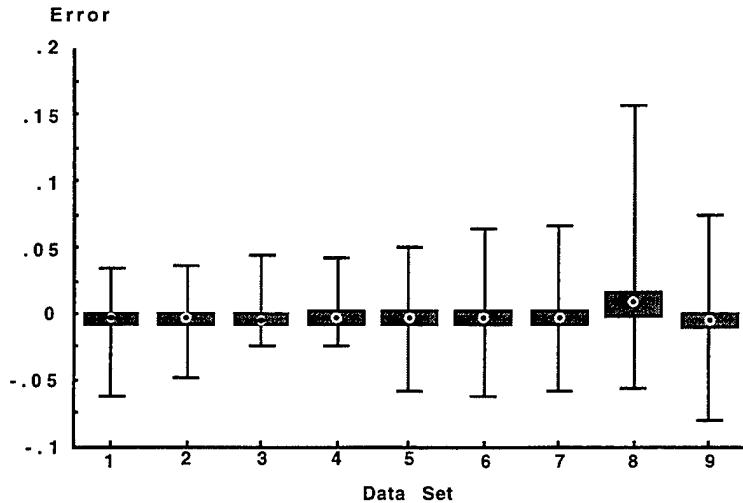


FIGURE 4. Box and whisker plot of errors;  $k=1, 2, \dots, 9$ .

We chose as a statistic the distance between the point determined by the sample mean and standard deviation  $(\bar{x}_k, s_k)$  and the origin. Recall that the origin  $(0,0)$  corresponds to perfect agreement between calculation and experiment. The distances

$$\sqrt{\bar{x}_k^2 + s_k^2} \quad k = 1, 2, \dots, 9 \quad (4)$$

were determined, and the ranking induced under this procedure in terms of the index  $k$  is as follows:

$$4 \prec 3 \prec 2 \prec 7 \prec 5 \prec 6 \prec 1 \prec 9 \prec 8. \quad (5)$$

Expression (5) asserts that the set of computed data corresponding to  $k=4$  is in closest agreement with the experimental measurements; data set  $k=3$  is second closest, ..., data set  $k=8$  is furthest away.

In Figure 5, the sample mean and standard deviation  $(\bar{x}_k, s_k)$  for each of the 9 data sets are plotted by case number  $k$ . More complicated ranking procedures are, of course, possible. An obvious modification would be to form a weighted combination of mean and standard deviation, but to do so without a compelling reason serves no practical purpose, and was not undertaken for these data.

### 2.3 Summary

The results of the direct ranking procedure are altogether consistent with the preliminary graphics. This is reassuring. A surprise at this stage usually means that an error in logic or calculation is still at large. As mentioned earlier, and as seen in Figure 2, the flowfield predictors seem to experience the most difficulty for larger circumferential values, so focusing on that subregion rather than the entire flowfield may be more appropriate.

For the statistician, failure to account for error in the experimental measurements  $Y_{ij}$  is troublesome. The statistician's model for experimental data is usually of the form: *observed*  $Y_{ij} =$  *actual*  $Y_{ij} +$  *error*  $e_{ij}$ . Engineering experience leaves the fluid dynamicist convinced that concern

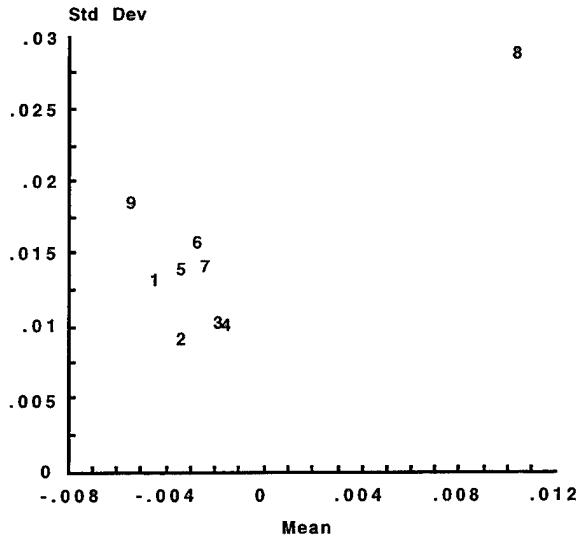


FIGURE 5. Scatterplot of mean and standard deviation for error sets  $k=1, 2, \dots, 9$ .

over this point is unwarranted. Moreover, in this instance, only a single set of experimental data is available and all the flowfield predictors are deterministic, so the error, whatever its magnitude, remains forever confounded with the recorded observations.

### 3. APPLICATION TO TTCP DATA

The approach developed in section 2 was applied to output from four turbulence models: the Baldwin-Lomax(BL), Baldwin-Barth(BB), k-omega(kw), and k-epsilon(ke), all using the NPARC computational code (Cooper and Sirbaugh 1989). The theory and implementation of these models is well documented and is apart from our considerations here.

The difficulty of visual comparison between experiment and computation over the four models is convincingly illustrated in Figure 6. Only four axial stations are shown, but, even from that small number, the difficulty facing the engineer attempting to rank the procedures is readily apparent.

The statistical approach was applied to the entire data sets: 41 circumferential locations at each of 14 axial stations for the 4 turbulence models. The box and whisker plot visualization is shown in Figure 7. The BL result has the shortest whiskers, indicating the smallest excursion from the experimental data. The mean error for all the data sets is close to zero, with little apparent advantage for one turbulence model over another. The results are shown in a scatterplot format in Figure 8. Again, the BL model seems to have the advantage; however, as shown in Table 1, the level of disagreement between the four turbulence models is very small.

The formal ranking is  $BL \prec kw \prec ke \prec BB$ , but, importantly, the small numerical differences listed in Table 1 hold substantial practical significance for the engineer. The four turbulence models vary widely in complexity and computer resource requirements. The most computer efficient model is the algebraic model BL, followed by the BB model. The two-equation models, kw and ke, are more complex and require significantly greater computer resources than either the BB or BL model.

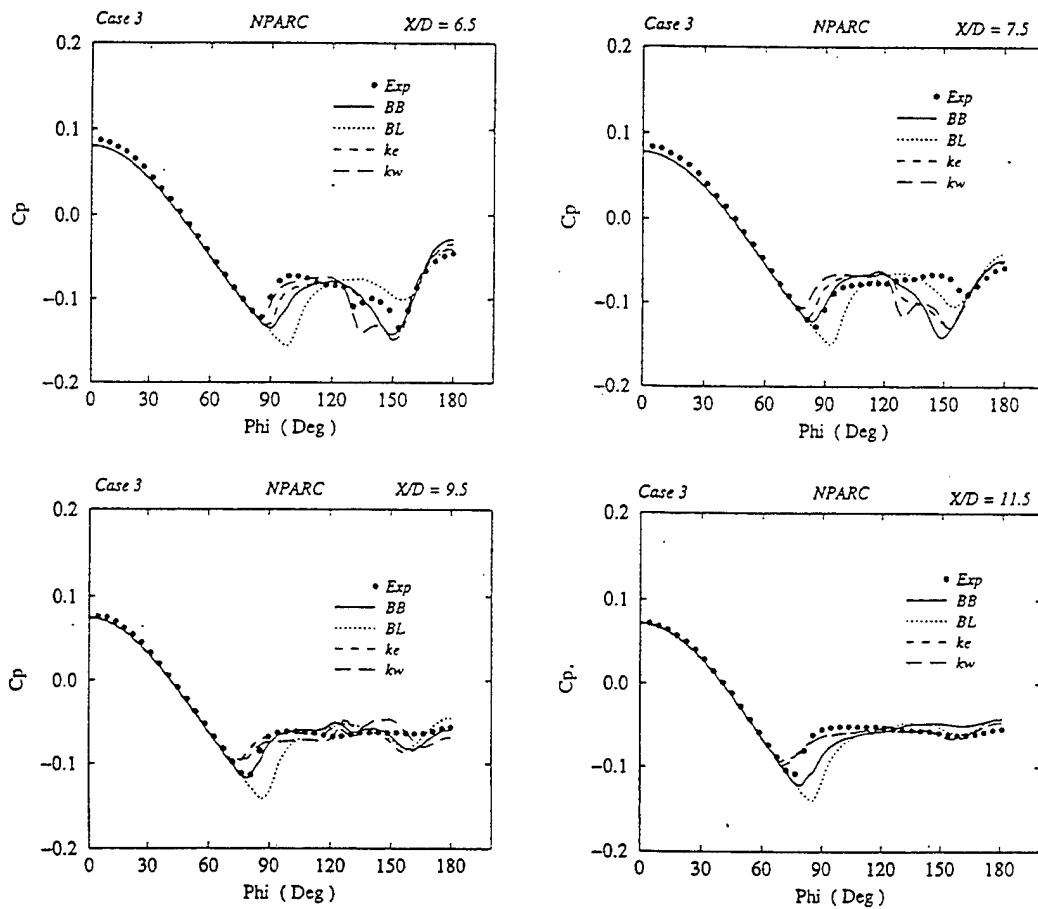


FIGURE 6. Surface pressure  $C_p$  vs. circumferential position  $\phi$ ; Mach 2.5, angle of incidence 14-deg. Comparison between computation and experiment for both laminar and turbulent viscous modeling.

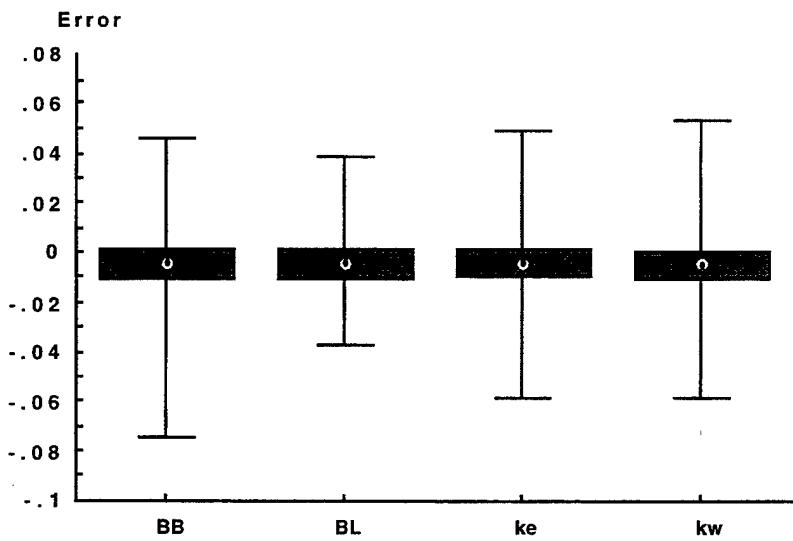


FIGURE 7. Box and whisker plot of errors for BB, BL, ke, and kw turbulence models; Mach 3.5, angle of incidence 14 deg.

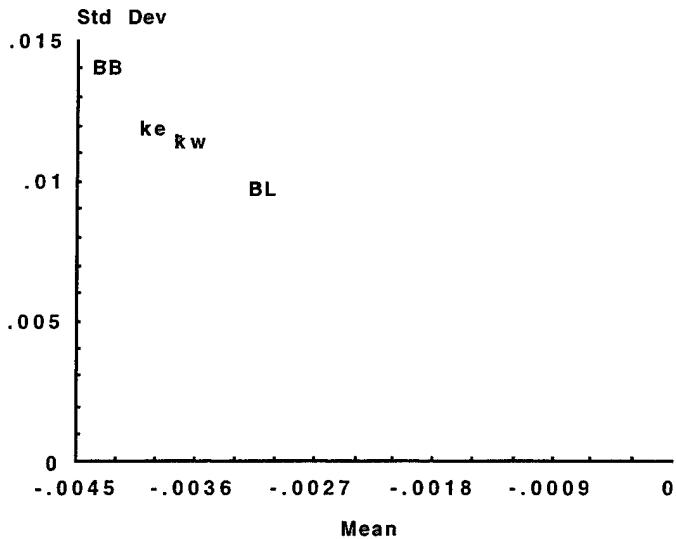


FIGURE 8. Scatterplot of errors for BB, BL, ke, and kw data sets; Mach 3.5, angle of incidence 14 deg.

TABLE 1. Distance Measurements for  $(\bar{x}, s)$

case:	BB	BL	ke	kw
distance:	0.01257	0.01125	0.01149	0.01131

#### 4. CONCLUSION

A statistical approach, with emphasis on data visualization, has been developed to assist the engineer in the comparison of computer models intended to predict surface pressure about a physical model in a complex flow field. The approach was applied to the evaluation of computational results obtained from four distinct turbulence models and showed that the use of more sophisticated computer-intensive turbulence models did not provide correspondingly superior results. The statistical analysis procedure extends directly to higher dimensions and should be particularly valuable when a large number of data sets are to be compared, and when an impartial quantitative assessment is sought.

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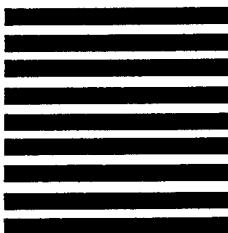
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